Statistical Physics

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The Microcanonical Ensemble

This represents the possible states of an isolated system whose total energy is exactly specified.

Number of states. $\Omega(E) =$ number of states with energy E; have $p(n) = \begin{cases} \frac{1}{\Omega(E)} & \text{if energy } = E(|n\rangle) \\ 0 & \text{otherwise} \end{cases}$

Entropy. $S(E) = k_b \log \Omega(E)$ (additive)

Temperature. $\dfrac{1}{T} = \dfrac{\partial S}{\partial E}$

Heat capacity. $C = \dfrac{\partial E}{\partial T}$ and $\Delta S = \displaystyle\int_{T_1}^{T_2} \frac{C(T)}{T} \, \mathrm{d}T$

Pressure. $p = T \left(\dfrac{\partial S}{\partial V} \right)_E$

First Law of Thermodynamics. $\mathrm{d}E = T\mathrm{d}S - p\mathrm{d}V$

Heat capacity (in terms of S). $C_V = T \left(\dfrac{\partial S}{\partial T} \right)_V$ and $C_p = T \left(\dfrac{\partial S}{\partial T} \right)_p$

The Canonical Ensemble

This represents the possible states of a system in thermal equilibrium with a heat reservoir at fixed temperature.

Approximation for $\Omega(E_{\mathbf{tot}})$. $\Omega(E_{\text{tot}}) \approx e^{S_R(E_{\text{tot}})/k_B} \displaystyle\sum_n e^{-E_n/k_B T}$

Boltzmann distribution for CE. $p(n) = \dfrac{e^{-E_n/k_B T}}{\sum_m e^{-E_m/k_B T}}$

Partition function. $Z = \displaystyle\sum_n e^{-\beta E_n} \implies p(n) = \frac{e^{-\beta E_n}}{Z}$ where $\beta = 1/k_B T$

Energy statistics for CE. $\langle E \rangle = -\dfrac{\partial}{\partial \beta} \log Z$, $\Delta E^2 = -\dfrac{\partial}{\partial \beta} \langle E \rangle = k_B T^2 C_V$

CE entropy. $S = -k_B \displaystyle\sum_n p(n) \log p(n)$ and $S = k_B \dfrac{\partial}{\partial T}(T \log Z)$

Free energy. $F = E - TS$, $F = -k_B T \log Z$ (most likely E minimises F)